# USE OF ACCESSIBILITY IN METROPOLITAN STRATEGIC PLANNING\*

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#### ABSTRACT

The conceptual and experimental development of a relationship between accessibility and density, together with utility measures which can be derived from it, is summarised. This relationship and its utility measures can be used to illustrate or describe aspects of the transport-traffic-land use system, to examine the suitability of development proposals in terms of efficiency and equity and to evaluate alternative transport proposals. Examples are presented and discussed. Finally, by a suitable combination of transport supply, travel demand and accessibility/density functions, points of stable equilibrium in the transport-traffic-land use system can be determined both before and after some disturbance to the system. The utility measures allow the value of the disturbance to be assessed. A radically different approach to the transportation planning process is therefore suggested and this is discussed, using work done in Canberra as an example.

## INTRODUCTION

1. The distribution of land uses, the movement efficiency of the transport system with respect to land use, and the stability of land use are important questions in metropolitan strategic planning. Work recently published by the author (Davidson 1977), provides an analytical basis for addressing these questions in such a way that only data already available from transportation studies are necessary.

2. Traditionally, transportation analyses have played a passive role in metropolitan strategic planning in that they were used to test the adequacy of transport networks which were postulated to meet the 'needs' of the land use pattern proposed. They may also have been used to check such values as the vehicle distance or vehicle time travelled in the metropolitan area for each strategy alternative, but such values could not, in any objective way, be compared to other qualities of each alternative.

3. The establishment of a relationship between the provision of transport and the land use effects of such provision begins to allow transport analyses to be used in a positive way.

## **CONCEPTUAL BACKGROUND**

4. A mathematical outline of the concepts reported in Davidson (1977) is given in Appendix A. A descriptive outline follows.

5. Accessibility is a quality, first defined by Hansen (1959), which can be described for each zone and reflects the view at the zone of the land use pattern and the transport system. If the transport system elements

serving the zone are improved, or if the activities relevant to a zone increase, then the zone's accessibility increases. The definition requires the use of an impedance function which describes the reaction of travellers to increasing travel cost.

6. Centrality, defined first for a specific impedance function by Patton and Clark (1970), is an inverse function of accessibility and is a kind of average travel time from the zone to all activities to which centrality is measured.

7. The author has extended the original work of Patton and Clark (1970) and shown that there is a linear relationship between centrality to employment and the logarithm of development density, i.e. of the combined density of population and employment. *Fig. 1* illustrates the relationship.



Fig. 1 — Density-centrality relationship. Analyses so far suggest that the intercept K, maximum possible density, is a constant whilst slope  $\beta$  is a function only of city size

8. A rationale for this relationship is that residents see accessibility and spacious surroundings as two conflicting objectives. The relationship represents the general way in which equilibrium is reached between the two objectives. Alternatively, if location costs are regarded as the sum of position rents and transport costs, then the density-accessibility relationship can be regarded as a line of equal total location costs.

9. Initial experimental results suggested that the value of the intercept K might be constant for all cities (i.e. there is a tendency towards the same maximum density in all cities) and that the slope of the line,  $\beta$ , was related only to city size.

10. If it is contended that this line represents an equilibrium position to which all zones in the city are tending (there is some experimental evidence for this type of conclusion in Patton (1970) ) then some important conclusions follow.

<sup>\*</sup>ACKNOWLEDGEMENTS: The author acknowledges the permission of the Commissioner, National Capital Development Commission, to publish this paper. However, the views expressed, particularly those related to the interpretation of Canberra analyses, are those of the author and not necessarily those of the Commission.

- (a) Since centrality is a function of both the general land use pattern and the transport system, then any change in either will affect the equilibrium density of all zones. In particular, a change to the transport system anywhere will have a measurable effect on the equilibrium density of zones everywhere.
- (b) Knowledge of the density and centrality of an area will allow a determination of how far that area is away from having an equilibrium density and since the tendency will be to move towards that equilibrium, an area's distance from the line will be a measure of the pressure for land use change in the area.\*
- (c) Because of the above conclusion, the effect of a specific transport improvement can be quantitatively estimated.
- (d) Knowing the centrality of an area also allows an appropriate density for strategic planning purposes to be set. If the planned densities are significantly different, then it can be assumed that they can be maintained only by firm control of strong market forces. For example, a forced density higher than equilibrium could lead to difficulty in selling houses or blocks or the need to reduce prices; conversely, planning for too low a density could lead to speculation and hence pressures for more intensive development.

11. By utilising the work of Neuburger (1971) and Koenig (1975) it is possible to show that, when city size remains constant<sup>†</sup>, centrality is a pure measure of disutility, i.e. it is linearly related to the value of properties. Hence, a decrease in centrality (i.e. of this form of average travel time) leads to a proportional increase in the locational component of property values. The economic effect on property of a change in the transport system or the distribution of employment can thus be measured.

12. It follows from the above that the difference between an area's actual centrality and the centrality experienced by an area in equilibrium at the same density is also a measure of another kind of utility. The utility in this case has been called density utility and is the utility derived from living in an area at a density below equilibrium (or the disutility from living at a density above equilibrium). This utility is a transient value and it is property owners seeking to realise this utility by redevelopment which causes the pressure for change which results eventually in equilibrium being reached.

13. The relationship between locational disutility and density utility is shown in *Fig. 2*. These two types of utility cannot be directly added because their conversion to money values would require different (so far undetermined) multiplication factors for each.

14. Centrality or locational disutility at a zone can be said to be experienced by all residents in the zone. Therefore, a total value for the zone can be obtained by multiplying by the population of the zone. The metropolitan total locational disutility  $-U_L$  is obtained



Fig. 2 — Derivation of density utility from density-centrality relationship

by adding all zonal total values. Hence, the metropolitan mean value,  $-\overline{u_L}$  is obtained by dividing the metropolitan total by the total population. Similar calculations can be done for density utility, yielding  $U_D$  and  $\overline{u_D}$  respectively.

15. It may be of interest to separate zones which have positive development pressures (i.e. have too low a density and hence a positive value of density utility) from those with negative development pressures (with too high a density and negative density utility values). This would enable a check to be made on whether a  $\overline{u_p}$ value close to zero was the result of zones generally being near equilibrium density or simply a cancellation of many underdeveloped zones, giving a high average value of positive density utility,  $(+u_D)$ , and many overdeveloped zones giving a high average value of negative density utility,  $(-u_D)$ . Standard deviations (or any other statistic) can be calculated for all of the mean values discussed. These give a measure of the variation of values throughout the metropolitan area.

16. Finally, average values of the utility measures experienced by any defined segment of the population may be calculated by multiplying each zonal utility value by the number of people in the zone qualifying for the segment, adding all zone totals, and dividing by the metropolitan population in the segment.

17. These results may be applied in metropolitan strategic planning to examine land use-transport elements in the following ways.

- (a) The total or metropolitan mean value of location disutility,  $-U_L$  or  $-\overline{u}_L$  respectively, is a joint measure of the efficiency of the land use distribution and the transport system proposed for it. When comparing alternatives, an objective should be to minimise this disutility.
- (b) The equity effects of alternatives can be assessed by segmenting the population and calculating location disutility and density utility for each segment as discussed above. The segments may be, for example, low, medium and high income groups. These segmental disutilities may be expressed either as totals, in order to assess the general equity of the metropolitan plan, or as differences between alternatives so as to assess the equity impact of each

And will provide the measure for growth opportunities in each zone for a land use allocation model such as the intervening opportunities model proposed in Golding and Davidson (1970) should the study require prediction of the distribution of natural growth.

t The effect of varying city size is described in Appendix B.

alternative. A common policy objective would be reflected if the mean locational disutility for low income groups was near average and if the standard deviation for low income groups was lower than average (so as to ensure that there are not pockets of badly locationally disadvantaged low income groups):

- (c) If reasonable land use stability is sought, mean density utility should be near zero with a low standard deviation. A highly-positive value would suggest that planned average densities were too low or that the scale of the proposed transport system could be reduced. High values of average positive density utility,  $(+u_D)$  or negative density utility,  $(-u_D)$ would suggest significant areas where planned density was inappropriately low or high respectively, thus drawing attention to potential difficulties in maintaining the development strategy in respect of those areas without very difficult planning controls.
- (d) The value of centrality for each zone could be used, in conjunction with the density-centrality relationship, to suggest an appropriate zonal density for planning purposes. It may thus be used to suggest which presently non-urban areas would be most likely to be readily developed, which existing suburban areas could most easily be developed to higher densities, and which areas may become subject to decay. Alternatively, if it is specially desired to develop or redevelop certain areas, or prevent others from decay, then specific transport or employment location strategies could be devised and tested to see if they produced centrality values consistent with the desired land use densities.

The general model may also be applied to other policy analyses and in conjunction with land use allocation models. For example, the effect of systematic changes in transport costs on the pattern of land use and development can be assessed. A major increase in fuel price can be shown as a change in the centrality profile and hence the profile of equilibrium densities. This will indicate the direction and strength of land use changes which might be expected to follow the fuel price increase and so guide in the formulation of policies on fuel, land use and transport.

# CANBERRA — A CASE STUDY

18. As part of a review of Canberra's metropolitan structure, analyses based on the above concepts are being carried out.

19. Since 1967, Canberra has been developed on the basis of what has been known as the 'Y' Plan which was proposed in National Capital Development Commission (1967). This plan proposed Canberra developing as a series of semi-separate towns with populations ranging between 50 000 and 150 000, each with its own town centre and other employment areas and together forming a Y shape. The concept is shown in *Fig. 3*.

20. At the present time Inner Canberra, Woden-Weston Creek and Belconnen have been more-or-less fully developed and major development is proceeding in the northern part of Tuggeranong which forms the base of the Y. However, the recent reductions in Canberra's growth rate and the general down-turn in economic activity have resulted in the development of the proposed Tuggeranong Town Centre being delayed. Thus, present residents of Tuggeranong, and those locating there in the near future, have very little local employment, and travel between home and work is significantly longer than expected when the growth strategy was formulated.

21. Therefore, one of the major issues in the metropolitan structure review is to determine whether the growth of Tuggeranong should continue, or whether more emphasis should be placed on developing the next new town of Gungahlin which would be the first town on the right arm of the Y. (Belconnen is the first town on the left arm of the Y and Woden-Weston Creek is between Canberra and Tuggeranong on the base of the Y.) Within this broad issue are questions about the appropriate distribution of employment, the densities of residential development, the details of the transport system and the relative costs of other infrastructure items and the whole range of less-easily quantified planning and environmental issues.

22. In order to illustrate the nature of those aspects of the review which are being addressed through an accessibility analysis, a summary of results for two of the initial alternatives is presented. One alternative, A, involves continuing with the planned development of Tuggeranong and completing it before Gungahlin is started. The other, B, proposes stopping Tuggeranong development as soon as practicable and proceeding instead with Gungahlin. Two stages have been calculated: a 430 000 population stage which could be reached after 1990, and a 300 000 population stage which could occur in the mid 1980s. For comparison, the 1975 situation (220 000 population) is also shown. Queanbeyan, across the border in N.S.W., is included in all analyses.

23. The calculations were done using inter-zonal travel times by car and by public transport for loaded networks thought to be appropriate to the alternative. (This, of course, is one of the matters to be tested.) Eighty-five per cent of accessibility by car and 15 per cent of accessibility by public transport were added together (to represent a 15 per cent mode split to public transport) and then added to intra-zonal accessibility. All accessibilities were calculated to employment. Zonal employment and population by income group were measured (for 1975) or estimated (for other stages). A power impedance function with n = 2.5 was used (see Appendix A). Figs 4 and 5 show computergenerated centrality contours at the 430 000 population stage for each alternative, whilst Table I shows the utility averages and standard deviations for 1975 and both time stages of both alternatives.

24. Considering first the  $u_L$  (metropolitan mean locational disutility or centrality) values, there is only a very minor difference between alternatives at the 1985 stage but a marked difference by 1995, with the Gungahlin alternative being superior. In fact, it is better on almost all counts and this is particularly clear by 1995:

- (a) Locational disutility is lower.
- (b) Variation (standard deviation) of locational disutility is lower.
- (c) Density disutility is lower and its variation is lower.
- (d) The average value of negative density utility is lower and by 1995 the standard deviation is lower.

N GUNGAHLIN C BELCONNEN NORTH CANBERRA 0 SOUTH CANBERRA WODEN WESTON  $\cap$ CREEK QUEANBEYAN GERNOM. KI UGGERANONG 2 С GENERAL PLAN CONCEPT (Y PLAN) /////// PARLIAM MANY 2011 URBAN AREAS INTERTOWN PUBLIC TRANSPORT 0 CINTHIS den as

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Fig. 3 — 1967 Canberra Metropolitan Strategy: the 'Y' plan

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Fig. 4 — Scheme A: completion of Tuggeranong Centrality contours at 430 000 population (1995?)

TABLE I

Fig. 5 — Scheme B: early development at Gungahlin Centrality contours at 430 000 population (1995?)

## AVERAGES AND STANDARD DEVIATIONS OF LOCATIONAL AND DENSITY DISUTILITY FOR EACH ALTERNATIVE METROPOLITAN STRATEGY AT EACH STAGE

|  |        | Whole<br>Population | Low<br>Income<br>Group | Medium<br>Income<br>Group | High<br>Income<br>Group |
|--|--------|---------------------|------------------------|---------------------------|-------------------------|
| u <sub>1</sub> : mean locational                 | 1975   | 9.51 (N.A.)         | 9.14 (N.A.)            | 9.49 (N.A.)               | 9.71 (N.A.)             |
| disutility (Std. Dev.)                           | 1985A  | 10.71 (2.96)        | 10.47 (3.14)           | 10.83 (2.93)              | 10.81 (2.80)            |
|  | В      | 10.59 (2.93)        | 10.35 (3.10)           | 10.71 (2.90)              | 10.70 (2.78)            |
|  | 1995A  | 15.39 (4.49)        | 15.08 (4.71)           | 15.42 (4.39)              | 15.60 (4.35)            |
|  | В      | 13.82 (3.84)        | 13.86 (4.30)           | 13.77 (3.68)              | 13.86 (3.53)            |
| un : mean density                                | 1975   | 0.49 (N.A.)         | 0.10 (N.A.)            | 0.58 (N.A.)               | 0.69 (N.A.)             |
| disutility (Std. Dev.)                           | 1985A  | 0.18 (2.95)         | 0.03 (3.10)            | 0.31 (2.93)               | 0.19 (2.85)             |
|  | В      | 0.12 (2.95)         | - 0.03 (3.10)          | 0.24 (2.93)               | 0.13 (2.84)             |
|  | 1995A  | 2.92 (4.22)         | 2.60 (4.47)            | 3.01 (4.10)               | 3.09 (4.11)             |
|  | В      | 1.72 (3.55)         | 1.70 (4.00)            | 1.74 (3.38)               | 1.73 (3.29)             |
| $(-u_{D})$ : mean of                             | 1975   |                     | N.A.                   | N.A.                      | N.A.                    |
| negative density                                 | 1985A  | 1.37 (1.68)         | 1.36 (1.69)            | 1.43 (1.70)               | 1.33 (1.65)             |
| utility (Std. Dev.)                              | В      | 1.34 (1.66)         | 1.34 (1.67)            | 1.40 (1.67)               | 1.30 (1.63)             |
|  | 1995A  | 3.55 (3.40)         | 3.45 (3.41)            | 3.57 (3.35)               | 3.61 (3.44)             |
|  | В      | 2.51 (2.59)         | 2.66 (2.90)            | 2.48 (2.46)               | 2.43 (2.42)             |
| $(+u_D)$ : mean of                               | 1975   | N.A.                | N.A.                   | N.A.                      | N.A.                    |
| positive density                                 | 1985 A | 1.19 (1.62)         | 1.33 (1.77)            | 1.13 (1.58)               | 1.14 (1.53)             |
| utility (Std.                                    | В      | 1.22 (1.63)         | 1.37 (1.79)            | 1.16 (1.59)               | 1.17 (1.53)             |
| Dev.)  | 1995A  | 0.63 (1.32)         | 0.85 (1.57)            | 0.57 (1.24)               | 0.51 (1.16)             |
|  | В      | 0.79 (1.39)         | 0.96 (1.58)            | 0.74 (1.31)               | 0.70 (1.27)             |
| Mean locational                                  | 1975   | 9.51                | 9.14                   | 9.45                      | 9.71                    |
| disutility, stand-                               | 1985 A | 9.10                | 8.90                   | 9.21                      | 9.19                    |
| ardised for city                                 | В      | 9.00                | 8.80                   | 9.10                      | 9.10                    |
| size at 1975                                     | 1995A  | 10.92               | 10.70                  | 10.95                     | 11.08                   |
| standard   | В      | 9.81                | 9.84                   | 9.78                      | 9.84                    |
| (i.e. $U_L \left(\frac{E_1}{E_2}\right)^{0.4}$ ) |        |                     |                        |                           |                         |

2)

Note: A: develop Tuggeranong fully B: Stop Tuggeranong growth and develop Gungahlin.

- (e) The average value of positive density utility is marginally higher but not so much as to be likely to generate significantly more redevelopment pressure. Standard deviations are similar.
- (f) The high values of standard deviation for both positive and negative density utility in all cases indicate significant lack of equilibrium in many zones. This suggests a need to reconsider planned densities in subsequent detailed planning at the zonal level.
- (g) There are no equity problems with either alternative, i.e. utilities for each income group are similar.

25. In general, the locational disutility values standardised for city size (see Appendix B) show the 1985 condition to be the best, but, by 1995, both alternatives have higher disutility than existed in 1975.

26. In 1975, and for both alternatives in 1985, average densities seem about right (density utility is near zero) but, if the equilibrium expression is correct, both alternatives by 1995 seem either to be at too high density or to have a transport system which is inadequate in terms of travel time — particularly the Tuggeranong alternative (density disutility is significantly above zero).

27. The two alternatives discussed are broadly-based examples of a whole range of alternatives being considered. Within broad alternatives there is significant scope for fine tuning of employment distribution, population distribution and density (lot size, medium density areas, etc.) and the transport system. Each modification is tested in the way described above, but smaller changes would be expected. In such cases a detailed examination, similar to the above, for a smaller part of the city, may also be called for.

28. The geographical distribution of the utility variables is being examined in detail also and areas where proposed density is significantly out-of-phase with equilibrium density are being examined and appropriate corrective action (a combination of variations in employment distribution, transport facilities and lot size) formulated.

29. Only alternatives which fulfil a whole range of other planning objectives are being considered and, within the framework established by location and density utility values, other infrastructure costs and environmental and amenity issues are being evaluated.

#### COMMENT

30. This initial analysis has been carried out whilst recognising that a number of conceptual and procedural issues lack final resolution. This can be achieved only through wider experience with the technique.

31. The density-centrality equilibrium, which is a most important component of the analysis for density utility measures and equilibrium density estimates, has

not been finally determined and awaits further analysis of other cities.

32. All the analysis so far has utilised a power impedance function for travel cost. It is quite possible that a negative exponential function would be more appropriate and it is proposed to repeat the analysis on that basis. However, the form of the impedance function, has a basic effect on the value of centrality, as shown in eqn (2) in Appendix A, and also on the form of the equilibrium relationship. Data from other cities would also need to be analysed on the same basis to give some confidence to equilibrium parameters.

33. A mode split of 15 per cent to public transport has been assumed in all analyses although it is expected to be much higher by 1995. However, the public transport coded networks prepared so far do not reflect the improved quality of public transport which would be appropriate. Hence, it was considered that, at this stage, assuming a constant 15 per cent mode split would give the best estimate of the real situation. Public transport networks which reflect future higher usage will be prepared for later, more detailed, analyses.

34. No work has yet been done to relate these utilities to money values although it is believed that an equivalence could be determined from an examination of property values.

## CONCLUSIONS

It is submitted that analyses based on the con-35. cept of accessibility are very valuable tools in urban planning. These analyses provide a statement of equilibrium between land use and the transport system and allow the degree of divergence from the equilibrium to be calculated. Locational utility which measures the efficiency of the present or proposed transport and land use arrangements and density utility which, in measuring the divergence from equilibrium, provides an estimate of the distribution of the pressure for land use changes, are very useful concepts, capable of simplifying statistical manipulation. The transport-land use equilibrium allows either appropriate densities to be formulated or suggests the transport system and/or employment distribution changes necessary to restore equilibrium.

36. The analyses are very simple to perform, particularly for any city which has typical transportation study data. Modifications can be done by changing the coded transport networks for transport changes and the zone lists of employment and population for land use changes.

37. In adopting this approach, the author and his colleagues have, as transportation planners, found that it provides an excellent means of matching transport and land use proposals in a realistic and broadly based way. The National Capital Development Commission is finding that straight-forward numerical results are a very helpful way of comparing and evaluating alternative strategies in terms of their transport-land use components.

#### APPENDIX A

# MATHEMATICAL SUMMARY OF ACCESSIBILITY MODEL OF TRANSPORT-LAND USE EQUILIBRIUM

38. In Davidson (1977) the following are shown.

(a) If accessibility at zone i,  $X_i$  to activity type S of which there are  $S_i$  in zone j is defined as

$$X_{i} = \sum_{j} S_{j} f(c_{ij})$$
(1)

where  $C_{ij}$  = interzonal travel cost between *i* and *j* and f(c) = an impedance function, e.g.  $c^{-n}$  or  $e^{\lambda c}$  and if centrality<sup>\*</sup> at zone *i*,  $Y_i$ , is defined so that, irrespective of impedance function:

$$X_{i} = \left(\sum_{j} S_{j}\right) f(Y_{j})$$
(2)

then there is a relationship between D, a combination of population density  $(D_p)$  and employment density  $(D_E)$  and centrality such that

$$\ln (D) = K - \beta Y \qquad (3)$$

(b) Initial experimental results utilising data from London (see Davidson 1973), Brisbane and Mackay suggested that it may be possible to give values to K and  $\beta$  which mean that accessibility is absolutely related to density and that an adequate general model may be:

$$\ln (D_{\rho} + 0.6 D_{E}) = 12.65 - \frac{48}{E^{0.4}} Y$$
 (4)

$$= 12.65 - \frac{48}{x^{0.4}}$$
(5)

if the impedance function is  $c^{2.5}$ ,  $S_j$  is the number of jobs in zone *j*, *E* is the total metropolitan employment  $(\Sigma_j S_j)$  and if accessibility is calculated as a combination of intra-zonal accessibility (i.e. accessibility due to employment within the zone) and inter-zonal accessibility by car and by public transport.

(c) Extending the work of Neuburger (1971) and Koenig (1975) it can be shown that, if metropolitan size remains constant, centrality is a pure measure of disutility related to location and movement efficiency. This utility was called location utility  $u_L$  so that

$$Y_i = -u_{L_i} \tag{6}$$

(d) Assuming that the density-centrality relationship in eqn (2) above represents an equilibrium condition then another type of utility can be inferred from the distance a zone is from the equilibrium line. This was called density utility  $u_p$  and defined as (see Fig. 2)

$$u_{D} = \frac{K - \ln D}{\beta} - Y \tag{7}$$

This is a transient value, measuring the pressure for land use change. A positive value suggests the potential for redevelopment with the property owner able to capitalise the utility in the process of redevelopment up to the equilibrium line. A negative value suggests either over-development in fringe areas, or incipient decay in inner areas.

## **APPENDIX B**

# EFFECT OF CITY SIZE ON CENTRALITY-DISUTILITY EQUIVALENCE

39. Equating centrality with disutility required metropolitan size  $(\Sigma_j S_j = E_T)$  to remain constant. In order to compare situations which involve a change in the size of the city, the effect on utility must be determined.

40. Koenig (1975) and Neuburger (1971) both show, to use Koenig's words, that 'Utility is an increasing function of accessibility'. Furthermore, it has been shown that accessibility is related directly to the natural logarithm of equilibrium density in a way which is independent of city size.

41. On another approach it is reasonable to presume that if general metropolitan growth occasions no change to the equilibrium density of a particular zone in the city, then the locational utility of that zone must have remained unchanged. If utility had changed, pressures for land use changes beyond those which may have been needed to achieve the original equilibrium would have occurred. Yet if equilibrium density remains constant, then accessibility must have remained constant. This leads to the same conclusion as the previous paragraph.

- 42. Thus, it can be stated that:
- (a) utility is an increasing function of accessibility;
- (b) utility is an increasing function of density; and
- (c) the accessibility-density relationship determined conceptually and experimentally makes the first two statements synonymous.

Hence, with a negative exponential impedance function

$$X_i = \sum_j S_j e^{-\lambda C_{ij}}$$
(8)

$$= (\sum_{j} S_{j}) e^{-\lambda Y_{j}}$$
(9)

if 
$$\sum_{j} S_{j} = E$$
, then  $X = E e^{-\lambda Y}$  (10)

43. When growth occurs,  $E_1$  is the original metropolitan employment and  $E_2$  is resultant metropolitan employment. Constant utility occurs

Centrality was first defined, for an impedance function of c<sup>n</sup>, by Patton (1970). Eqn (2) is a generalised definition for any impedance function.

when X remains constant. But utility under constant size has been determined in terms of Y so to transform:

$$\ln X = \ln E - \lambda Y \tag{11}$$

$$\frac{1}{\lambda}\ln X = \frac{1}{\lambda}\ln E - Y$$
 (12)

44. Therefore

$$\frac{1}{\lambda} \ln E_1 - Y_1 = \frac{1}{\lambda} \ln E_2 - Y_2$$
 (13)

is the requirement for constant utility. Hence,  $Y_2$  and  $Y_1$  represent the same utility when

$$Y_2 = Y_1 + \frac{1}{\lambda} \ln \frac{E_2}{E_1}$$
 (14)

This can also be derived directly from Neuburger's and Koenig's work.

45. Similarly, with a power impedance function

$$X_i = \sum_i S_j c_{ij}^{-n}$$
(15)

$$X = E Y^{-n} \tag{16}$$

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- $\frac{1}{X^{1/n}} = \frac{1}{E^{1/n}} Y$ (17)
- The accessibility-density relationship related log density to  $1/X^{lh}$  whilst utility has been defined in terms of Y.

46. Hence 
$$\frac{Y_1}{E_1^{1/n}} = \frac{Y_2}{E_2^{1/n}}$$
 (18)

when  $E_1$  and  $E_2$  represent different city sizes and  $Y_1$ and  $Y_2$  the centralities of a zone whose equilibrium density has not changed. Thus, to convert relative utilities described in centrality terms for a constant city size into measures which can be compared when size changes, all measures must be divided by  $1/E^n$ . Alternatively, if the size represented by  $E_1$  is regarded as the base, other utilities derived from centralities at size represented by  $E_2$  may be normalised with respect to the base by multiplying by  $(E_1/E_2)^{1/n}$  when a power impedance function is used.

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