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Accessibility in transport/land-use modelling and assessment

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Abstract. The relationship between accessibility and urban density is examined both conceptually and experimentally. A linear relationship between the logarithm of density and centrality, a derivative of accessibility, is calibrated. It is shown that centrality can be used to measure the utility of location in the context of the land-use/transport system. This provides a basis for evaluating land-use/transport changes by using only data readily available from transportation studies.

Transportation planning has suffered from a shortage of models which indicate the effect on land use of proposed transport-system changes. Thus the regular 'four-step' model (generation, distribution, mode-split, assignment) has no satisfactory closure between the output transport system and the input future land use; transport investment alternatives have not been explicitly evaluated for land-use impacts; the opportunity to use transport investments as an instrument of land-use policy has usually been foregone; and satisfactory analytical indications of the relative values of alternative metropolitan transport/land-use strategies have not been forthcoming.

This paper contains a way of addressing the relationship between accessibility and land-use intensity which provides a basis for a land-use model and generates a utility measure for those elements of the urban system specifically susceptible to influence by transport investments or general development decisions.

In section 1 of the paper the concept of accessibility is discussed and a hypothesis on the nature and operation of its relationship with land-use intensity presented. Work done to calibrate such a relationship is described in section 2. In section 3 the existence of the relationship is assumed and utility measures are developed from it. Applications of the relationship and the utility measures are discussed. Section 4 contains comment, application, and conclusions.

1.1 Accessibility and its relationship with land use

The concept of accessibility was first stated by Hansen (1959) as being

$$X_i = \sum_j S_j \mathrm{f}(c_{ij}) \,, \tag{1}$$

where

X_i is the accessibility at zone i,

S_j is a measure of activity at zone j, and

c_{ij} is a measure of cost of interaction between zones i and j, such that $\mathrm{f}(c_{ij})$ is the measure of travel impedance between i and j.

Weibull (1976) developed those axioms which he considered necessary for a satisfactory accessibility measure, and Hansen's general form as stated above is appropriate. However, one axiom is so formulated that a power impedance function for $\mathrm{f}(c_{ij})$ is excluded. But Weibull admits, when proposing that axiom, that its particular requirements are not basic but somewhat arbitrary.

A powerful aspect of the accessibility concept is that it combines in a single, simple measure the relevant characteristics both of the land use and the transport system. Thus any change in either system will, in general, lead to a change in accessibility at every point within the area of the system. Furthermore, the accessibility measure utilises only data that are already available through the normal data analysis of a transportation study.

Despite its potential power, accessibility, as a measure in its own right, has tended to be regarded almost as a curiosity. In Hansen's formulation it can be identified as a balancing factor in constrained gravity models and, in various forms, it has sometimes been used as a variable in trip-generation and mode-split models. Regional analysts sometimes calculate a variation which they call 'potential'.

It is now increasingly recognised that transport is one of the factors which affect land-use distribution and intensity. If this is so, then it follows that a change in transport will, of itself, tend to generate changes in land use. Since expenditure on transport is one of the largest elements of public capital expenditure, it would seem to be important to seek to understand as much as possible about the community effects of such expenditure in the interests both of maximising its effectiveness and of trying to achieve as many public objectives as possible by the expenditure.

There is no question that land-use decisions are influenced by many considerations other than transport, but that is not necessarily relevant if the concern is with the effect of transport changes on land use. Accessibility is a useful concept for the study of such an effect since it has a single value at each location and, properly calibrated, will change appropriately with changes in the transport system. In building up a model concept, it could be postulated that one objective of location is to maximise accessibility. This, however, must be balanced by an opposing effect. The equilibrium between these two effects could then be the relationship between accessibility at a point and a measure of land-use intensity at that same point. The following hypothesis identifies the opposing effects:

Hypothesis Each individual in an urban system seeks both space for his private life and closeness to others and to activities which are either of interest or necessary to his well-being. He is repelled by very close activities and attracted to more distant ones. He thus seeks to minimise the density at which he lives and to maximise his accessibility.

This notion implies that a balance must be struck between two conflicting desires. The way in which individuals strike that balance, when taken collectively, provides the accessibility—density profile, which should always be tending towards equilibrium. It follows that a change in accessibility will introduce instability, which will tend to be corrected by development or redevelopment in the case of accessibility increase, or decay in the case of decrease. Hence knowledge of the relationship will allow impacts on land use at a particular location to be inferred from any change in either the transport system or the land-use pattern.

It is stressed that this hypothesis represents a two-dimensional cut through a multi-dimensional space, in that many other factors are also taken into account in locational choice. Nevertheless accessibility and density are explicitly manageable and specifically relevant to transportation decisions. Many of the other factors are not. Furthermore, an understanding of the accessibility-density relationship may provide a framework within which the influence of other factors may be studied.

It is proposed therefore to investigate a relationship of the type,

$$\phi(\text{land-use intensity}) = \text{accessibility},$$

or, to be more specific,

$$\phi(D_i) = X_i = \sum_j S_j \mathrm{f}(c_{ij}) \,, \tag{2}$$

where $\phi(D_i)$ is a function of a general measure of density in zone i.

Aggregate values are used in the relationship both because they are the data most commonly available and because there is value in keeping to purely spatial (as against disaggregate behavioural) measures which are robust and independent of the detailed composition of the population. In this way the relationship can provide a basis for broad regional or metropolitan analysis(1).

1.2 The accessibility measure

1.2.1 *Available modes of travel*

The measure of accessibility should take into account the travel opportunities presented by all modes in such a way that, if an extra mode is introduced without disturbing existing modes, the overall accessibility should increase. Thus an approach adopted by some workers in determining accessibility of utilising only the minimum cost by any mode between each zone pair is inappropriate. A weighted sum of accessibility by each mode is required.

1.2.2 *Intrazonal accessibility*

Accessibility calculations are often subject to significant variation with zone size, generally because the accessibility of each zone to itself has not been included. A general measure for intrazonal accessibility by public transport is likely to be very difficult to derive but various possibilities exist for a combination of car and walk travel. Intrazonal accessibility is $S_i \mathrm{f}(c_{ii})$, so if a measure of intrazonal travel cost c_{ii} can be determined an all-modes single measure of intrazonal accessibility can be calculated.

1.2.3 *Total accessibility*

Taking account of both of these points, the composite total accessibility is defined as,

$$X_i = X_i^{\mathrm{I}} + \gamma X_i^{\mathrm{T}} + (1-\gamma) X_i^{\mathrm{C}} \ , \tag{3}$$

where

X_i^{I} is the intrazonal accessibility,
X_i^{T} is the interzonal accessibility by public transport,
X_i^{C} is the interzonal accessibility by car, and
γ is a parameter to be determined.

1.2.4 *Impedance function*

The impedance function, $\mathrm{f}(c_{ij}^{\mathrm{k}})$ for each mode k may be either the exponential form $\exp(-\lambda c_{ij}^{\mathrm{k}})$ or the power form $(c_{ij}^{\mathrm{k}})^{-n}$. It should be the same as that used in the gravity distribution model and should have similar parameter values. Generalised interzonal cost, c_{ij}^{k}, could be taken as interzonal travel time or a more complex cost measure if it is available [for example in determining public transport accessibility, c_{ij}^{T} (the T superscript identifies cost as being via the public transport mode) could have weighted components reflecting in-vehicle time, waiting, walking and transfer time, fare, and comfort; for private transport, components of c_{ij}^{C} (where superscript C identifies cost as being for car travel) could be travel time, perceived vehicle-operating costs, parking charges, and walking time].

1.2.5 *Measure of activity*

Accessibility can be determined to any measurable activity, S_j. It could be, for example, retail floor space, hospital beds, or unskilled employment. For our purpose we need a measure which is a surrogate for the activities which people in general consider important when making location decisions. Pragmatically it could be regarded

(1) Measures of accessibility which are used in travel-demand models, particularly those associated with disaggregate data, have no such need to be restricted to spatial variables and may be constructed in quite different ways, for example, as suggested by Ben-Akiva (1977).

as the measure which produced the best relationship between accessibility and density. It could also perhaps be developed from Wilson's (1967) form of gravity model:

$$T_{ij} = a_i P_i b_j A_j \mathrm{f}(c_{ij}) \,, \tag{4}$$

$$a_i = \frac{1}{\sum\limits_j b_j A_j \mathrm{f}(c_{ij})} = \frac{1}{X_i} \qquad \text{if} \quad S_j = b_j A_j \,. \tag{5}$$

S_j could then be thought of as a transport-system-independent measure of attraction-type activities, since A_j is the actual number of trip attractions and b_j is a modifier which, with a_i, the trip production modifier, produces the fully constrained model. A transformation of $a_i P_i = V_i$ would lead to the same form as the totally unconstrained gravity model,

$$T_{ij} = V_i S_j \mathrm{f}(c_{ij}) \,, \tag{6}$$

where V_i and S_j were pure measures of trip producing and trip attracting activity respectively. In this context V_i and S_j could be postulated to be functions of combinations of quantifiable indicators of size, which could be calibrated to be the best values of

$$V_i = \mathrm{f}(\text{employment}_i, \text{population}_i, \text{etc}) = a_i P_i \,, \quad \text{and}$$

$$S_j = \mathrm{f}(\text{employment}_j, \text{population}_j, \text{etc}) = b_j A_j \,.$$

V_i and S_j would then be rooted entirely in land-use parameters rather than in the combined transport and land-use parameters of $a_i P_i$ and $b_j A_j$. Thus it is assumed that the impact of transport on land use is discernable in the land use alone if the right measures can be found.

The question of an appropriate measure of activity was explored experimentally to some extent (see section 2) and employment was adopted. The deficiencies of this measure are explored later.

1.3 The density measure

The density measure needs to reflect the intensity both of residential and of other uses of land, taken together. If the problem is simplified by assuming that urban land is used either for residence or for employment, and that all other land is beneficially regarded as open space, then density can be measured in terms of persons employed and residents. If it is further assumed that there is a number of workers at a workplace which has the same effect as one resident on other residents' perception of the spaciousness of their surroundings, then density D can be defined as,

$$D = D^{\mathrm{P}} + \alpha D^{\mathrm{E}} \,, \tag{7}$$

where

D^{P} is the density of population

D^{E} is the density of employment, and

α is a parameter to be determined.

1.4 The functional form of the relationship

Clark C (1951) proposed a relationship between residential density and distance from the city centre, r, as

$$D^{\mathrm{P}} = a \exp(-\beta r) \,, \tag{8}$$

which could be transformed to

$$\ln D^{\mathrm{P}} = \ln a - \beta r = K - \beta r \,. \tag{9}$$

Clark's model did not work near the city centre, where residential density declined, but replacing Clark's D^{P} with D (total density) should correct that. A relationship of this nature appears to be reasonable for a city with a dominant city centre. The influence of the motor car, particularly since Clark produced his paper, seems to have changed the nature of cities so that they tend to be decreasingly monocentric. Wilbur Smith (1961) produced density-radius profiles for a number of American cities which demonstrate a reducing conformity with exponential decay up to 1960. Bellomo et al (1970) showed that Detroit, a city particularly influenced by the car, had a distinct middle-distance density plateau in 1953 which simply increased in height by 1965. Beard and Oxlad (1969) showed that Brisbane corridors lost their exponential decay of density with radial distance in several corridors between 1933 and 1947, and that by 1966 a well developed density plateau existed which was tending to increase in height over time.

If Clark C was right in 1951 with his proposals regarding density and radial distance in monocentric cities, then this suggests that the same relationship may hold for average distance (cost) to activities in all cities.

Two ways are proposed to generate an average cost from accessibility and these have been called 'centrality', developed by Patton and Clark N (1970), and 'average travel cost'.

Patton and Clark N, who derived centrality from an accessibility formulation which used the power impedance function, c_{ij}^{-n}, defined it as:

$$\text{centrality, } Y_i = \left(\frac{\sum_j S_j}{\sum_j S_j c_{ij}^{-n}}\right)^{1/n} = \left(\frac{\sum_j S_j}{X_{(n)i}}\right)^{1/n} , \tag{10}$$

where $X_{(n)i}$ is accessibility calculated from a power impedance function with exponent n, and regarded it as a means of normalising accessibility so it could be used across different total populations.

A more useful way to regard centrality (its use is made clear in section 3) and a more general definition is obtained from,

$$X_i = \sum_j S_j \mathrm{f}(c_{ij}) = \left(\sum_j S_j\right) \mathrm{f}(Y_i) , \tag{11}$$

where f( ) refers to the same function on both sides of the equation. Thus the form of the impedance function determines the relationship between centrality and accessibility, and centrality is the travel cost to be applied to the whole area if it is regarded as one zone[(2)].

Average travel cost is derived from the unconstrained gravity model. By the normal definition of an average,

$$\text{average travel cost, } M_i = \sum_j c_{ij} T_{ij} \Big/ \sum_i T_{ij} . \tag{12}$$

(2) It can be seen that the Patton and Clark N definition is a special case of the general definition in equation (11) with $\mathrm{f}(c_{ij}) = c_{ij}^{-n}$, and therefore $\mathrm{f}(Y_i) = Y_i^{-n}$. With a negative exponential impedance function, $\mathrm{f}(c_{ij})$ and $\mathrm{f}(Y_i)$ are respectively $\exp(-\lambda c_{ij})$ and $\exp(-\lambda Y_i)$, and

$$X_{(\lambda)i} = \sum_j S_j \exp(-\lambda c_{ij}) = \left(\sum_j S_j\right) \exp(-\lambda Y_i) , \tag{11a}$$

so

$$Y_i = \frac{1}{\lambda} \ln\left(\frac{\sum_j S_j}{X_{(\lambda)i}}\right) . \tag{11b}$$

In the unconstrained gravity model, again using a power impedance function, $T_{ij} = P_i S_j c_{ij}^{-n}$, so

$$M_i = \frac{\sum_j c_{ij} P_i S_j c_{ij}^{-n}}{\sum_j P_i S_j c_{ij}^{-n}} = \frac{P_i \sum_j S_j c_{ij}^{1-n}}{P_i \sum_j S_j c_{ij}^{-n}} = \frac{X_{(n-1)i}}{X_{(n)i}} \,. \tag{13}$$

Hence from equation (9), substituting D for D^{P}, the proposed functional form of the accessibility-density relationship is either

$$\ln D_i = K - \beta Y_i \tag{14}$$

$$\left[\text{with a power impedance function, } \ln D_i = K - \beta \left(\frac{\sum_j S_j}{X_{(n)i}}\right)^{1/n}\right],$$

or

$$\ln D_i = K - \beta M_i \tag{15}$$

$$\left[\text{with a power impedance function, } \ln D_i = K - \beta \frac{X_{(n-1)i}}{X_{(n)i}}\right].$$

2.1 Calibration of an accessibility-density relationship

Detailed analyses of transportation study data for London and Brisbane were undertaken to clarify and calibrate an accessibility-density relationship. The work undertaken in London has been separately reported (Davidson, 1973) but will be summarised here.

For no better reasons than that gravity model work was almost entirely done with power impedance functions in the author's environment at the time this work was first developed, and that the pioneering work by Patton and Clark N (1970) used power functions, the accessibility work was calibrated using such a function. Hence the relationship to be calibrated was

$$\ln(D_i^{\mathrm{P}} + \alpha D_i^{\mathrm{E}}) = K - \beta \left(\frac{\sum_j S_j}{X_{(n)i}}\right)^{1/n} , \tag*{(3) (16a)}$$

where

$$X_{(n)i} = X_{(n)i}^{\mathrm{I}} + \gamma X_i^{\mathrm{T}} + (1-\gamma) X_i^{\mathrm{C}} \,, \tag{16b}$$

and

$$X_{(n)i}^{\mathrm{T}} = \sum_{j \neq i} S_j (c_{ij}^{\mathrm{T}})^{-n} \tag{16c}$$

$$X_{(n)i}^{\mathrm{C}} = \sum_{j \neq i} S_j (c_{ij}^{\mathrm{C}})^{-n} \tag{16d}$$

$$X_{(n)i}^{\mathrm{I}} = \mathrm{f}(S_i, \text{area}_i, \text{an average intrazonal travel cost}) \,. \tag{16e}$$

Values for K, β, n, α, γ had to be determined, a satisfactory measure of S devised and a means of measuring $X_{(n)}^{\mathrm{I}}$ developed. The basic equation (16a) was amenable to calibration by linear regression but only K and β could be determined directly. All other parameters had to be estimated by trial and error with the objective of maximising the coefficient of determination, R^2, in the regression. Interzonal travel times from the skim trees of the assignment models were used for c_{ij} values. Density was in terms of activity per square mile.

[3] Or the expression in equation (15).

2.1.1 *Intrazonal accessibility*

The method used to calculate intrazonal accessibility, $X^{\mathrm{I}}_{(n)}$, is not central to the argument and is described in the appendix. It is sufficient to say here that accessibility was computed to individual activities which were assumed to be evenly distributed throughout the zone, either by walking or by car depending on the travel times involved (car travel was assumed to involve a terminal time and hence for short journeys was slower than walking). In very dense zones in the city centre, the nature of the impedance function meant that intrazonal accessibilities were very high, in some cases higher than the total interzonal component. This has the not unreasonable imputation that central, CBD, locators are more interested in the activities immediately around them than those anywhere else.

2.1.2 *Activity measure*

Activity measures tested were employment, population, trip attractions, and trip productions. Employment produced the best results, and modifying the equation to include population by including a $\left(\sum_j P_j/X_i^{\mathrm{P}}\right)^{1/n}$ factor, calculated from the population, in the regression equation resulted in the same R^2 value as when it was excluded. Hence it was excluded.

2.1.3 *Calibration of parameters*

Both London and Brisbane gave values of $n = 2 \cdot 5$ and $\alpha = 0 \cdot 60$. For London, $\gamma = 0 \cdot 55$ and for Brisbane $\gamma = 0 \cdot 35$. In both cases this approximated to the proportion of metropolitan travel by public transport in the respective cities at the time the data was collected (1962 in London and 1968 in Brisbane). There was thus a fortuitous physical analogy between the proportion of accessibility to be determined from public transport and the metropolitan-wide use of public transport. This should suggest a value of γ for other cities.

Without undue constraint, all the regression lines could be made to pass through the value of 12·65 on the vertical axis. Hence $K = 12 \cdot 65$ for both cities, and the implication is that the maximum attainable density $(D^{\mathrm{P}} + 0 \cdot 6D^{\mathrm{E}})$ was $\exp(12 \cdot 65)$ or 312000 per square mile. If all of this was employment, it would be at a density of 520000 per square mile.

With K held at 12·65 the values of β were 0·115 for London and 0·283 for Brisbane. If equation (16a) is rewritten as

$$\ln D_i = K - \beta\left(\sum_j S_j\right)^{1/n}\left(\frac{1}{X_{(n)i}}\right)^{1/n} = K - \rho\left(\frac{1}{X_{(n)i}}\right)^{1/n}, \tag{17}$$

then if $\beta\left(\sum_j S_j\right)^{1/n}$ $(= \rho)$ can be shown to be constant for both cities, a relationship between density and accessibility which is independent of the city emerges. With the measures used, ΣS is the total metropolitan employment, E^{tot}, which was 4049000 in London and 296000 in Brisbane. Hence the values of ρ were 50·54 and 43·69 respectively. A value of 48 was selected and tested against limited data from smaller cities. Although the points were more scattered, they showed no trend towards lower values. Hence there was no systematic reduction in the ρ value as cities became smaller and the value of 48 was adopted.

Given the vast differences between London, Brisbane, and the smaller cities (down to 20000 population) against which the expression was tested, it is tentatively suggested that a relationship between density and accessibility (which could have some generality) can be stated as,

$$\ln(D^{\mathrm{P}} + 0 \cdot 6D^{\mathrm{E}}) = 12 \cdot 65 - 48X_{(n)}^{-0 \cdot 4}, \tag{18}$$

where $n = 2 \cdot 5$.

Regression analysis gave R^2 values of 0·90 in the case of London and 0·76 in the case of Brisbane. This reflects the supposition discussed in section 1 that mature cities are likely to express any equilibrium between accessibility and density more than rapidly growing cities. Thus even if the suggested relationship is absolutely correct as an expression of equilibrium, it is an equilibrium which is never likely to be completely reached, and the magnitudes of divergence from it, which are reflected in the R^2 values in London and Brisbane, would be expected. This must mean that any examination of empirical evidence, as this has been, which seeks to find an equilibrium which is never in fact reached must end with some uncertainty about the quality of the equilibrium position derived. This analysis has at least resulted in errors which both in distribution and magnitude would be intuitively expected.

A linear relationship with mean travel cost, based on equation (15), also gave reasonable results, but a result in this form did not appear to be as useful as the centrality form and was not carried further.

London data and the travel time from Charing Cross were used to also check Clark C's original formula. Centrality and mean travel cost gave better results.

In centrality terms, equation (18) may be restated as

$$\ln(D^{\mathrm{P}}+0\cdot 6D^{\mathrm{E}}) = 12\cdot 65 - 48(E^{\mathrm{tot}})^{-0\cdot 4}Y, \tag{19}$$

where Y has been calculated using a power impedance function with $n = 2\cdot 5$.

Figure 1 shows London, Brisbane, and Mackay (Queensland) zones plotted against this centrality-density relationship.

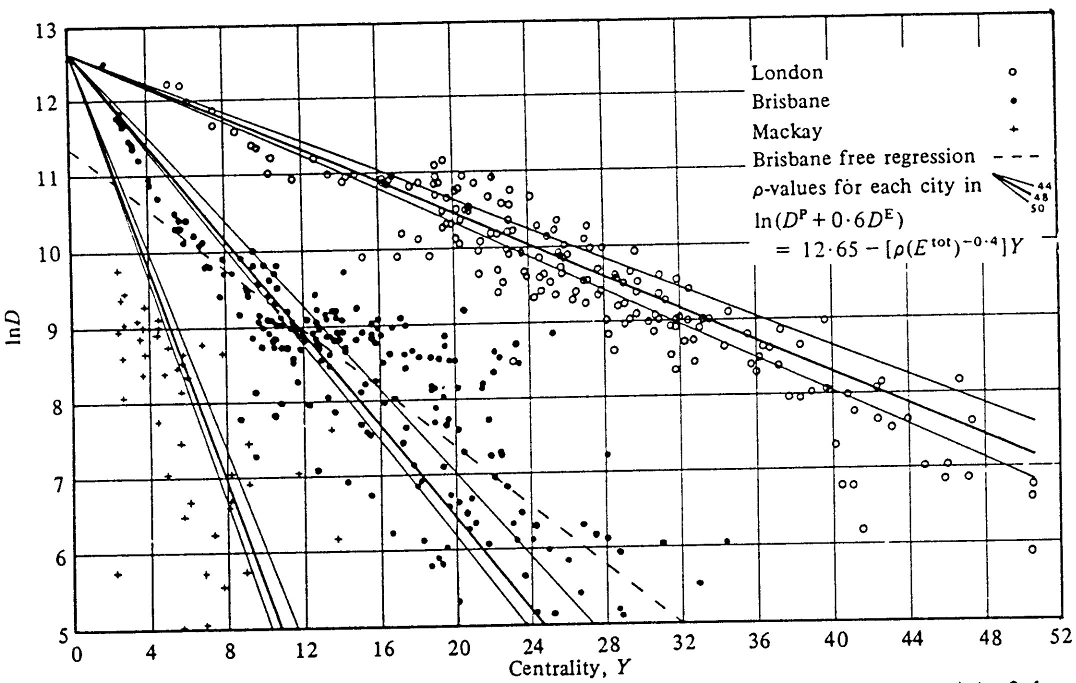

Figure 1. Centrality-density relationship of the form $\ln(D^{\mathrm{P}}+0\cdot 6D^{\mathrm{E}}) = 12\cdot 65 - [\rho(E^{\mathrm{tot}})^{-0\cdot 4}]Y$ for various cities.

3.1 A utility measure for evaluating alternative transport land-use plans

3.1.1 *Utility of location*

Neuburger (1971) showed that the change in consumer surplus resulting from a change in accessibility, when it is calculated with an exponential impedance function, is

$$\Delta S = \frac{1}{\lambda}\sum_i O_i \ln\frac{X_i^{(1)}}{X_i^{(2)}}\ , \tag{20}$$

where

$$X_i = \sum_j S_j \exp(-\lambda c_{ij}) . \tag{21}$$

Neuburger, using classical consumer surplus theory, developed this from considerations of the gravity model and the statement

$$\Delta S_i = \sum_j \int_{c_{ij}^{(1)}}^{c_{ij}^{(2)}} T_{ij} \mathrm{d}c_{ij} . \tag{22}$$

Koenig (1975), using microeconomic theory, developed "an appropriate variable describing the desirability of potential destinations" and found that "the expected utility, u_i, derived from trips by a resident of zone i is an increasing function of ... Accessibility". Specifically, using the definition of accessibility given in equation (21) he found that

$$u_i = \frac{1}{\lambda} \ln X_i + \text{constant} . \tag{23}$$

Therefore

$$\Delta u_i = \frac{1}{\lambda} \ln \frac{X_{1i}}{X_{0i}} . \tag{24}$$

Koenig's statement has the opposite sign to Neuburger's with Koenig's X_0 (Neuburger's X_1) as the original condition and X_1 (X_2) as the changed condition. In what follows, u_i is a relative utility for an individual in i, U_i is the total relative utility for a zone ($= u_i P_i$), and a minus sign indicates disutility.

Thus the increase in utility for an individual within a zone, when accessibility increases from X_0 to X_1, is

$$\Delta u_i = \frac{1}{\lambda} \ln \frac{\sum_j S_j \exp(-\lambda c_{1ij})}{\sum_j S_j \exp(-\lambda c_{0ij})} . \tag{25}$$

The concept of centrality as stated in equation (11) allows this to be taken much further. Equation (25) may be rewritten as

$$\Delta u_i = \frac{1}{\lambda} \ln \frac{\left(\sum_j S_j\right) \exp(-\lambda Y_{1i})}{\left(\sum_j S_j\right) \exp(-\lambda Y_{0i})} \tag{26}$$

$$= Y_{0i} - Y_{1i} , \tag{27}$$

and

$$u_i = -Y_i + \text{constant} . \tag{28}$$

Hence, with constant city size, centrality is a direct measure of the disutility to an individual of a location with that centrality. The addition of the constant provides for all the other factors which contribute to the total utility of any site. The expression $-Y = u^{\mathrm{L}}$ gives the relative utility, which depends only on the location of the site in the context of the transport/land-use system.

The author has so far found the transformations required to repeat the above derivations with a power impedance function to be mathematically intractable. However, the fact that centrality is a direct measure of utility allows centrality derived from a power impedance function to be similarly used.

This result is conceptually credible, as centrality has the units of travel cost and is a kind of weighted average travel cost to all activities.

3.1.2 *Utility arising from density*

The postulated relationship between centrality and density allows a further component of total utility to be stated in centrality terms and hence to be evaluated when testing transport/land-use alternatives.

Consider a zone in a city which has a centrality Y and a density D (see figure 2). The calibrated density–centrality relationship (for example that developed in section 2) which will be of the form,

$$Y = \frac{K - \ln D}{\beta} , \tag{29}$$

suggests that at density D, residents are prepared to accept a centrality of Y'. The fact that they enjoy the lower centrality (relative disutility), Y, suggests that they have an added utility, above what people generally are prepared to accept at that density, of $Y' - Y$.

Thus the utility created only by the local density, u^{D} (derived by living at density D when centrality is Y) is

$$u^{\mathrm{D}} = \frac{K - \ln D}{\beta} - Y . \tag{30}$$

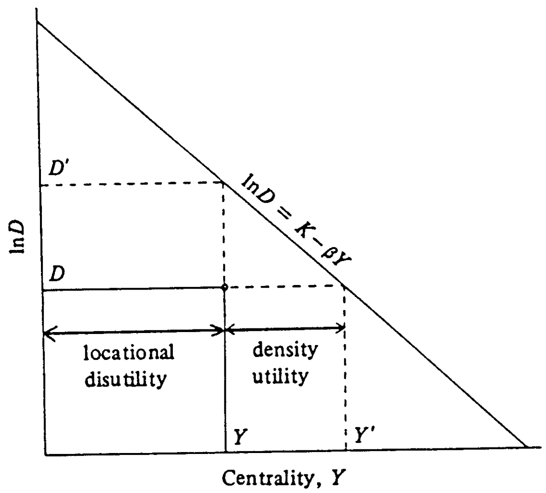

Figure 2. Derivation of density utility from centrality–density relationship.

3.1.3 *Transport-related utility*

Utility derived from density, u^{D}, cannot be directly added to the utility derived from location, u^{L} $(= -Y)$, because the unit values of each will be different and have not been determined. If the unit values are such that ω units of u^{L} are equivalent to one unit of u^{D}, a relative value of total utility which specifically recognises the two effects can be stated as

$$u^{\mathrm{D+L}} = -\omega Y + \left(\frac{K - \ln D}{\beta} - Y\right) + L , \tag{31}$$

where

ω is an unknown parameter, and

L is an unknown constant.

For purposes of comparison of two transport/land-use alternatives, L can be eliminated but ω cannot. Hence

$$\Delta u = \frac{1}{\beta} \ln \frac{D_0}{D_1} + (1 + \omega)(Y_0 - Y_1) . \tag{32}$$

Again it must be emphasised that urban residents would perceive the utility to be derived from a particular residence in much more complex terms than just the surrounding density and the location with respect to transport and the rest of the urban area. However, these two elements are those most affected by the impact of the transport system and are measurable in terms of transport/land-use parameters, and so should be sufficient for an assessment of transport impacts.

A definitive answer requires the estimation of ω and, for some purposes, L. Although no work has been done by the author on these problems, it is suggested that an examination of property values may lead to a solution. If property values are regarded as a capitalisation by the market of residential utility, then the value of $u^{\mathrm{D+L}}$ should be reflected in these values. Hence if categories of properties are selected so that the internal qualities of properties (that is those characteristics not related to location and surrounding density) are similar for each category, then the differences in values within each category should be linearly related to the relative utility of location and density. An expression,

$$\text{value} = \sigma\left\{\frac{K-\ln D}{\beta} - (1+\omega)Y + L\right\} , \tag{33}$$

would be valid. Value, D, and Y are variable, K and β are known constants, and σ, ω, and L are parameters to be determined. The form of the expression is such that the unknown parameters could be estimated by linear regression. In this form it is likely that L would vary between categories but it would be desirable if ω were constrained not to do so.

3.1.4 *Area total and average utility*

To analyse alternative metropolitan transport/land-use schemes, total or average values of transport related utility (that is, owing to location and density) for each scheme need to be determined. For this comparison measure, L can be eliminated since it does not vary directly as a result of changes in the transport/land-use system. The utility for residents would be obtained by weighting each zonal utility value either by the population or by number of households in each zone. If population is selected, then

$$U^{\mathrm{P}} = \sum_i P_i\left\{\frac{K-\ln D_i}{\beta} - (1+\omega)Y_i\right\} , \tag{34}$$

U^{P} is the transport-related total utility of residents in the metropolitan area for a transport/land-use scheme with density distribution D_i and centrality values Y_i. The mean value per resident, $\bar{u}^{\mathrm{P}}$, is

$$\bar{u}^{\mathrm{P}} = \sum_i P_i\left\{\frac{K-\ln D_i}{\beta} - (1+\omega)Y_i\right\} \Big/ \sum_i P_i , \tag{35}$$

Determination of the transport-related utility of economic activities (or firms) in a metropolitan area under alternative schemes is more difficult. Simple weighting factors such as employment or floor space do not seem to be adequate. In any case the original hypothesis of this paper is not necessarily true for firms, although in section 2 it was calibrated with a total density which included employment density. As a first attempt, estimates could be made by substituting zonal employment for population in equations (34) and (35).

3.1.5 *Use of experimentally determined relationship*

If the relationship developed in section 2 is used for utility calculations, then in equations (30) to (35), $K = 12{\cdot}65$ and $\beta = 48(E^{\mathrm{tot}})^{-0{\cdot}4}$, where E^{tot} is total employment in the study area. For example, area total utility created by density can then be directly calculated from

$$U^{\mathrm{D}} = \sum_i P_i \left(\frac{12 \cdot 65 - \ln D_i}{48 (E^{\mathrm{tot}})^{-0 \cdot 4}} - Y_i \right) . \tag{36}$$

Applications, comments and conclusions

4.1 Applications

The accessibility-density relationship provides a firm link between the transport system and its effect on land use. The fact that this can be expressed in utility terms opens the way for evaluation of individual projects or a whole metropolitan system. The fact that utility can be attributed to individual areas or socioeconomic groups means that the equity effects of proposals can be determined.

4.1.1 *Land-use modelling*

The likely existence of an equilibrium between accessibility and density has been demonstrated. Work by Patton (1970) showed that, after accessibility changes, density tended to move towards the new equilibrium point. The nature of the utility expression emphasises this fact in that, when density is below the equilibrium level, locators will be attracted to take advantage of the extra utility ($Y' - Y = u^{\mathrm{D}}$ is positive) available, whereas when density is above the equilibrium level, residents will tend to move out because of their existing low level of utility (u^{D} is negative). Hence the relationship can be used directly to indicate the nature and potential scale of the impact of land-use changes which result from changes in the transport system. In transportation study terms, the relationship can help determine whether a proposed transport system is in sympathy with, and tends to reinforce, the land-use pattern it has been designed to serve.

Where it is desired actually to predict land-use changes in a growth situation without any institutional constraints, then the relationship needs to be used in conjunction with an allocation model. For example a form of intervening opportunities model will allocate a given amount of growth among zones if the ranking of zones and the available growth capacity in each zone is determined. Difficulties with both of these requirements have limited the use of this approach, but the accessibility-density relationship can be used to determine them satisfactorily. The zones can be ranked by $|u^{\mathrm{D}}|$, since density utility is a transient value the realisation of which provides the impetus for change towards equilibrium, and the potential growth (or decline) can be obtained directly from the relationship. In terms of figure 2, the zones are ranked in order of $|Y' - Y|$ and the opportunities available are $D' - D$. This approach is an extension of that suggested in the intervening opportunities land-use model developed by Golding and Davidson (1970), the use of which was reported by Grigg et al (1972).

Development thresholds could be estimated from equation (33) if parameters ω and L were known. For development to occur, property values must be at least equal to the cost of serviced land at the density proposed plus the construction cost of each house. The threshold value of Y at which development at a certain density is just feasible can be determined by solving for Y in equation (33), in which the development cost is substituted for value, and D is the proposed development density.

4.1.2 *Evaluation of alternative projects or systems*

The determination of transport-related utility in terms of readily calculated parameters means that alternative transport projects or systems can be evaluated in terms of their metropolitan effects on travel costs and land use. If ω and σ in equation (33) can be determined, then changes in transport-related utility can be used directly in the evaluation. Otherwise separate values of u^{L} and u^{D} can be computed for each alternative, and various qualitative assessments made of the impact of each alternative on aspects of the metropolitan system. The size and distribution of Δu^{D} will indicate the dimensions of the land-use impacts of the proposal, since it is the capitalisation or sale of u^{D} which leads to land-use changes towards the equilibrium.

4.1.3 *Studies of equity*

Equation (35) is an expression for determining mean metropolitan relative utility arising from location and density effects. Similar results can be obtained for any segment of the population, and any of the usual statistics which may be considered useful could also be determined. Hence if say,

$$P_i = P_i^{(1)} + P_i^{(2)} + P_i^{(3)} , \tag{37}$$

where $P_i^{(1)}$, $P_i^{(2)}$, $P_i^{(3)}$ were, for example, numbers of people from respectively low, medium, and high income families in zone i, then

$$\bar{u}_{P^{(k)}} = \sum_i P_i^{(k)} \left\{ \frac{K - \ln D_i}{\beta} - (1+\omega)Y_i \right\} \bigg/ \sum_i P_i^{(k)} , \tag{38}$$

and $\bar{u}_{P^{(1)}}$, $\bar{u}_{P^{(2)}}$, and $\bar{u}_{P^{(3)}}$ could be compared for conformance with any policy on equity. The distribution of utility in any segment could be checked by determining the standard deviation of utility. Again this could be important in respect of policy. These calculations could be done in terms either of total values, as in equation (38), or of changes between schemes. The former approach would highlight any equity problem, whereas the latter would illustrate the equity impact of alternatives.

4.1.4 *Metropolitan strategic planning*

Although recognising that a wide range of issues are important in metropolitan strategic planning, equation (34) or (35) can be used to determine a single utility measure for each alternative plan which reflects the efficiency of the transport/land-use system and the distribution of densities. In these terms, the objective would be to maximise total or mean utility. The distribution of centrality gives a good indication of what would be an appropriate distribution of densities and would highlight any areas where densities are proposed to be grossly too high or too low.

4.2 Comments and suggestions for further work

A distinction should be drawn between section 2 and the remainder of the paper. In the bulk of the paper the initial hypothesis is developed and the uses of the results which flow from it are described. Some experimental work which suggests that the hypothesis may be accepted is presented in section 2.

Much more experimental work needs to be done and it needs to be done in a wide range of cities. It should look at different density measures, it should be done for an exponential impedance function and it should seek to include in the measure of activity, S, more than just employment.

In regard to this last point, it is intuitively clear that residents and firms are influenced in their location by the location both of other residents and of other firms. It is unlikely that the distribution of employment is an adequate surrogate of this. Indeed, the author's earlier work demonstrated that accessibility to population is distributed in quite a different way than is accessibility to employment, with the former peaking some distance from the city centre. The effect in London is reported by Davidson (1973), and Martin and Dalvi (1976) have reported similar results for London. To incorporate population would complicate the relationship but probably is worthwhile. It might also make it more reasonable to calculate the locational and density utility of firms if the accessibility–density relationship incorporated a population factor in accessibility.

The relationship between transport-related utility and property values could usefully be explored, as its calibration would allow location and density utility to be compared and may lead to acceptable money values for those utilities.

Finally, utilities have been aggregated not only within segments but across the whole metropolitan area. Despite the economic impropriety of such an approach, it is submitted that it is necessary and appropriate to do so in a metropolitan-scale planning instrument.

5.1 Conclusion

This paper has presented a hypothesis which implies that for each value of accessibility there is an equilibrium density and that areas in cities tend to move towards that equilibrium. This notion has been shown to have some experimental justification. Its existence creates significant opportunities in the transportation planning process, particularly in the area of land-use modelling. The results which flow from it, together with the transformation of accessibility into utility, lead to improved ways of assessing transport projects and testing the utility and equity of important aspects of metropolitan strategy plans.

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APPENDIX

Determination of intrazonal accessibility

Intrazonal accessibility is determined at the centroid of the zone. Assume that activities to which accessibility is being calculated are uniformly distributed throughout the zone and that the zone is circular with the same area as the real zone. Accessibility is thus calculated at the centre of the circle, and all travel is assumed to be radial. Assume further that the modes available are car and walking and that the choice between them is made so as to minimise travel time. Use of a car will incur a terminal travel time whereas walking will not.

If t is terminal time (min), s is average car speed (km h^{-1}), and w is walking speed (km h^{-1}), then there will be a radius, r_o, at which it is equally fast to walk or drive. Then

$$\frac{r_o}{w} = \frac{r_o}{s} + \frac{t}{60} , \tag{39}$$

whence

$$r_0 = \frac{wst}{60(s-w)} . \tag{40}$$

The radius, R, of the zone of area A, is, of course, $(A/\pi)^{1/2}$. The distance from the centre to the nearest activity, when these activities are at density D, is $D^{-1/2}$.

There are thus two distinct areas to which accessibility is to be calculated: an annulus with radii $D^{-1/2}$ and r_o, which is reached by walking at a radial speed of w_i, and an annulus with radii r_o and R, which is reached by driving from the centre at a radial speed of s after taking a terminal time t.

An annular ring with inside radius r and width δr will have an area of $2\pi r\delta r$ and so will have $2\pi r\delta rD$ activities within it.

The distance accessibility of this area to the centre is $2\pi r\delta rD/r^n$ with a power impedance function. Hence accessibility of a ring with radii a and b is

$$X = \int_a^b \frac{2\pi D}{r^{n-1}}\,\mathrm{d}r . \tag{41}$$

Integration of this expression gives, in general:

$$\left.\begin{aligned} X &= \frac{2\pi D}{n-2}\left(\frac{1}{a^{n-2}} - \frac{1}{b^{n-2}}\right) , && n \neq 1, 2 , \\ X &= 2\pi D(b-a) , && n = 1 , \\ X &= 2\pi D(\ln b - \ln a) , && n = 2 . \end{aligned}\right\} \tag{42}$$

For the walking area, $D^{-1/2}$ and r_o substitute for a and b, and distance accessibility is converted to time accessibility by multiplying the result by w^n.

For the area reached by driving, the algebra is complicated by the terminal time. Again it is convenient to derive the equations in distance terms so that the terminal time, t, is converted to a distance equivalent of L (= ts). Then accessibility to a narrow ring is $2\pi Dr\delta r/(L+r)^n$, for which the controlling integral is:

$$\left.\begin{aligned} \int \frac{r}{(L+r)^n}\,\mathrm{d}r &= \frac{(r+L)^{2-n}}{2-n} - \frac{L(L+r)^{1-n}}{1-n} , && n \neq 1, 2 , \\ &= r+L-L\ln(r+L) , && n = 1, \\ &= \ln(r+L) + \frac{L}{r+L} , && n = 2 . \end{aligned}\right\} \tag{43}$$

These integrals are determined between the limits of r_o and R and the results converted to time by multiplying by s^n.

Total intrazonal accessibility is the sum of accessibilities to the walking and driving zones and is:

$$\left.\begin{aligned} X^{\mathrm{I}}_{(n\neq 1,2)} &= 2\pi D\left[\!\!\left[s^n\left\{\frac{1}{2-n}\left[(R+L)^{2-n}-(r_o+L)^{2-n}\right]\right.\right.\right. \\ &\qquad \left.\left.\left. -\frac{L}{1-n}\left[(R+L)^{1-n}-(r_o+L)^{1-n}\right]\right\}+\frac{w^n}{2-n}\left[r_o^{2-n}-D^{-\frac{1}{2}(2-n)}\right]\right]\!\!\right] , \\ X^{\mathrm{I}}_{(n=1)} &= 2\pi D\left[\!\!\left[s\left\{R-r_o-L\ln\frac{R+L}{r_o+L}\right\}+w\left[r_o-D^{-\frac{1}{2}}\right]\right]\!\!\right] , \\ X^{\mathrm{I}}_{(n=2)} &= 2\pi D\left[\!\!\left[s^2\left\{\ln\frac{R+L}{r_o+L}-L\frac{R-r_o}{(R+L)(r_o+L)}\right\}+w^2\left[\ln r_o+\tfrac{1}{2}\ln D\right]\right]\!\!\right] . \end{aligned}\right\} \qquad (44)$$

In some small zones, driving never becomes faster and $r_o > R$. In these circumstances terms multiplied by s^n must be eliminated to avoid invalid results. Walking speeds, driving speeds, and terminal time can be separately specified for each zone to reflect local conditions. This calculation has been based on a power impedance function. A similar approach can be used for an exponential function $\exp(-\lambda r)$ which results in

$$\begin{aligned} X^{\mathrm{I}}_{(\lambda)} = 2\pi D &\left\{\exp\left[-(r_o+L)\frac{\lambda}{s}\right]\left(\frac{r_o s}{\lambda}+\frac{s^2}{\lambda^2}\right)-\exp\left[-(R+L)\frac{\lambda}{s}\right]\left(\frac{Rs}{\lambda}+\frac{s^2}{\lambda^2}\right)\right. \\ &\left. +\exp\left(-D^{-\frac{1}{2}}\frac{\lambda}{w}\right)\left(\frac{D^{-\frac{1}{2}}w}{\lambda}+\frac{w^2}{\lambda^2}\right)-\exp\left(-\frac{r_o\lambda}{w}\right)\left(\frac{r_o w}{\lambda}+\frac{w^2}{\lambda^2}\right)\right\} . \end{aligned} \qquad (45)$$